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Gravitational Waves and Chameleon Mechanism in F(R) Gravity

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References: "Gravitational Waves in F(R) Gravity: Scalar Waves and the Chameleon Mechanism" arXiv:1902.02494

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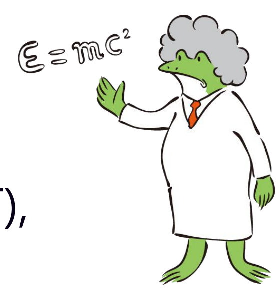


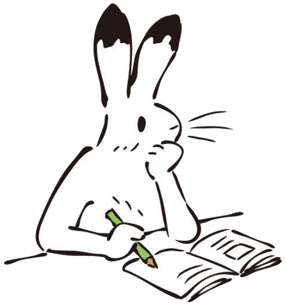
Discovery of Gravitational Waves (GWs)

Observation of GWs

- Consistent with GR predictions ``so far”
- Test of modified gravity (phenomenology)
- Limit on speed of (tensorial modes of) GW
- cf.) Constraint on Horndeski theory (G_4, G_5)

GWs in F(R) Gravity?

- For Dark Energy (DE), Cosmological Constant Problems
- Dynamical DE = New scalar field
- Additional polarization mode of GW
- Can we ``observe” it or not?
- Something new? (many papers in last decades…)



F(R) Gravity Theory

F(R) Gravity

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}F(R)$$

[Buchdahl (1970)] etc.

cf.) EH action

$$\int d^4x\sqrt{-g}R$$

To generalize: $R \to F(R)$

Equation of Motion

$$F_R(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}F(R) + \left(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right)F_R(R) = \kappa^2 T_{\mu\nu}$$

cf.) $F_R(R) = \partial_R F(R)$

Trace part of EOM

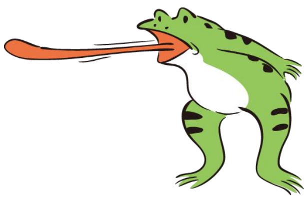
$$\Box F_R(R) = \frac{1}{3}\kappa^2 T + \frac{1}{3}\left[2F(R) - F_R(R)R\right]$$

New Scalar Field = Additional Polarization Mode of GW

More on Scalar Field

Scalar Field in (original) Jordan frame

Definition of Scalar Field



$$\Phi \equiv F_R(R)$$

Trace part of EOM = Scalar Field EOM

$$\Box\Phi = \frac{\mathrm{d}V(\Phi)}{\mathrm{d}\Phi} + \frac{1}{3}\kappa^2 T^{\mu}_{\ \mu}\,, \quad \frac{\mathrm{d}V(\Phi)}{\mathrm{d}\Phi} \equiv \frac{1}{3}\left[2F(R(\Phi)) - R(\Phi)F_R(R(\Phi))\right]$$

Eff. Potential

$$\frac{\mathrm{d}V_{\mathrm{eff}}}{\mathrm{d}\Phi} = \frac{1}{3}\left[2F(R) - RF_R(R) + \kappa^2 T^{\mu}_{\ \mu}\right]$$

$$2F(R_{\mathrm{min}}) - R_{\mathrm{min}}F_R(R_{\mathrm{min}}) + \kappa^2 T^{\mu}_{\ \mu} = 0$$

$$m_\Phi^2 = \left.\frac{\mathrm{d}^2 V_{\mathrm{eff}}(\Phi, T)}{\mathrm{d}\Phi^2}\right|_{\Phi=\Phi_{\mathrm{min}}} = \frac{1}{3}\left[\frac{F_R(R_{\mathrm{min}})}{F_{RR}(R_{\mathrm{min}})} - R_{\mathrm{min}}\right]$$

(Eff.) Mass changes according to external (matter) field

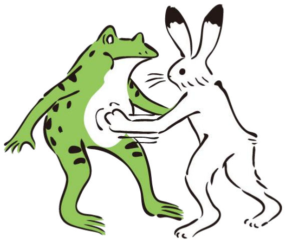
Heavy in high-density region（Chameleon Mechanism）

GWs are also ``Screened”?

Perturbations

Metric = 2 Tensor Modes

Scalar Field = 1 Scalar Mode



[Capozziello and Corda et. al (2008)] [Liang, Gong, Hou and Liu (2017)] etc.

Scalar Mode of GWs

Scalar Field = Background + Oscillation

Matter Distribution — Scalar Mode of GW

Chameleon Mechanism affects Scalar Mode?

- To depend on environment in which GWs propagate
- To create environment “by hand” (Prof. Nojiri’s talk)

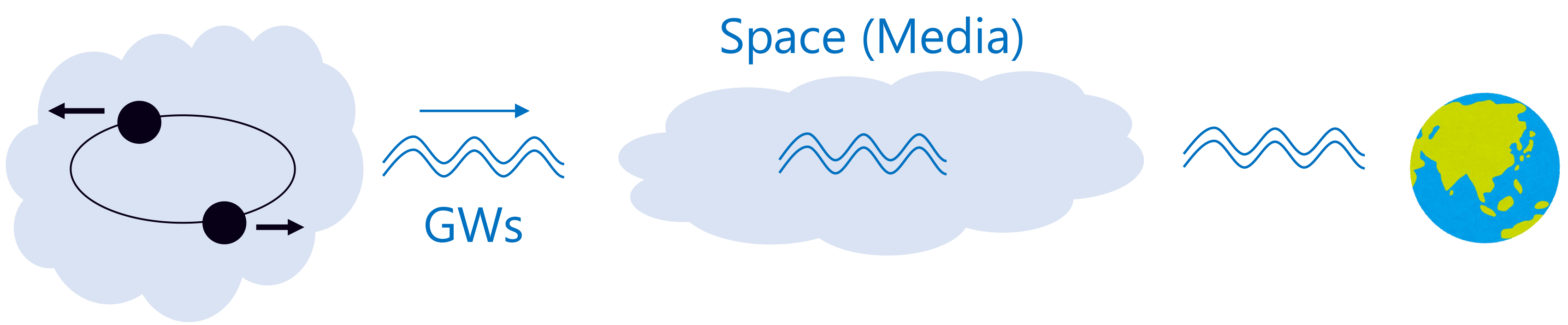
How to Deal with ``Environment Dependence"

Three possible situations...

Step 1: Emission [Jana and Mohanty (2018)] etc.

Step 2: Propagation [Lindroos, Llinares and Mota (2016)] etc.

Step 3: Detection (Our Target)



Focus on detection environment ($T^{\mu}{}_{\mu}$)

– We focus on relation b/w incident & penetrating waves

GWs in F(R) Gravity

Formulation with matter (for chameleon mechanism)

$$\Phi R_{\mu\nu} - \frac{1}{2}F(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\Phi = \kappa^2 T_{\mu\nu}(g^{\mu\nu}, \Psi)$$

Perturbations w.r.t. metric and scalar field

$$g_{\mu\nu} = b_{\mu\nu} + h_{\mu\nu}\,, \quad \Phi = \Phi_{\min} + \phi$$

Up to 1^{st} order

$$\begin{aligned}
&\left(\nabla^{(b)}_\mu \nabla^{(b)}_\nu - b_{\mu\nu}\Box^{(b)} - R^{(b)}_{\mu\nu}\right)\phi \\
&\quad = \Phi_{\min}\delta R_{\mu\nu} - \tfrac{1}{2}\Phi_{\min}\delta R b_{\mu\nu} - \tfrac{1}{2}F(R^{(b)})h_{\mu\nu} \\
&\qquad - \tfrac{1}{2}b_{\mu\nu}\left(2\nabla^{(b)}_\alpha h^\alpha{}_\rho - \nabla^{(b)}_\rho h\right)\nabla^\rho_{(b)}\Phi_{\min} - b_{\mu\nu}h^{\alpha\beta}\nabla^{(b)}_\alpha\nabla^{(b)}_\beta\Phi_{\min} \\
&\qquad + \tfrac{1}{2}\left(\nabla^{(b)}_\mu h_{\nu\rho} + \nabla^{(b)}_\nu h_{\mu\rho} - \nabla^{(b)}_\rho h_{\mu\nu}\right)\nabla^\rho_{(b)}\Phi_{\min}
\end{aligned}$$

N.B.) Matters are fixed ($\delta T_{\mu\nu} = 0$) as external field

(I want to) Simplify situation…

To consider maximally symmetric space-time

- Ricci scalar is constant cf.) Minkowski and (Anti-)de Sitter
- Background scalar field $\Phi_{\rm min}$ is also constant

Derivatives of $\Phi_{\rm min}$ vanish

$$\delta R_{\mu\nu} - \frac{1}{2}\delta R\, b_{\mu\nu} - \frac{1}{2}\frac{F(R^{(b)})}{\Phi_{\rm min}} h_{\mu\nu} = \left(\nabla^{(b)}_{\mu}\nabla^{(b)}_{\nu} - b_{\mu\nu}\Box^{(b)} - R^{(b)}_{\mu\nu}\right)\varphi$$

To redefine scalar field $\varphi \equiv \phi/\Phi_{\rm min}$

Trace part

$$\left(\Box - m_{\Phi}^2\right)\varphi = -\frac{1}{3}\Phi_{\rm min} h^{\mu\nu} R^{(b)}_{\mu\nu} + \frac{1}{6}F(R^{(b)})h$$

N.B.) Background = Environment $2F(R_{\rm min}) - R_{\rm min}F_R(R_{\rm min}) + \kappa^2 T = 0$

We Consider Following Model

Starobinsky model with R^2 correction

$$F(R) = R - \beta R_c \left[1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right] + \alpha R^2$$

where $R_c \sim \Lambda$ and $\alpha, \beta, n > 0$

GR

Viable F(R) gravity model for DE

[Starobinsky (2007)]

to cure singularity problem

[Frolov (2008)] [Kobayashi, Maeda (2008)] [Dev et al. (2008)]

Large-curvature limit ($R > R_c$)

$$F(R) \simeq R - \beta R_c + \beta R_c \left(\frac{R_c}{R}\right)^{2n} + \alpha R^2$$

where $\beta R_c \approx 2\Lambda$, $\beta \sim \mathcal{O}(1)$

Another parameter $\alpha < 2.3 \times 10^{22}\ [\mathrm{GeV}^{-2}]$

[Berry and Gair (2011)][Cembranos (2009)]

Around de Sitter Solution

Exact Minkowski is unstable $(m_\Phi^2 < 0)$

- ``Approximately'' flat ($R_{\mu\nu} \approx 0$), then, $F(R \approx 0) \approx 0$
- Evaluating scalar field mass with non-zero curvature

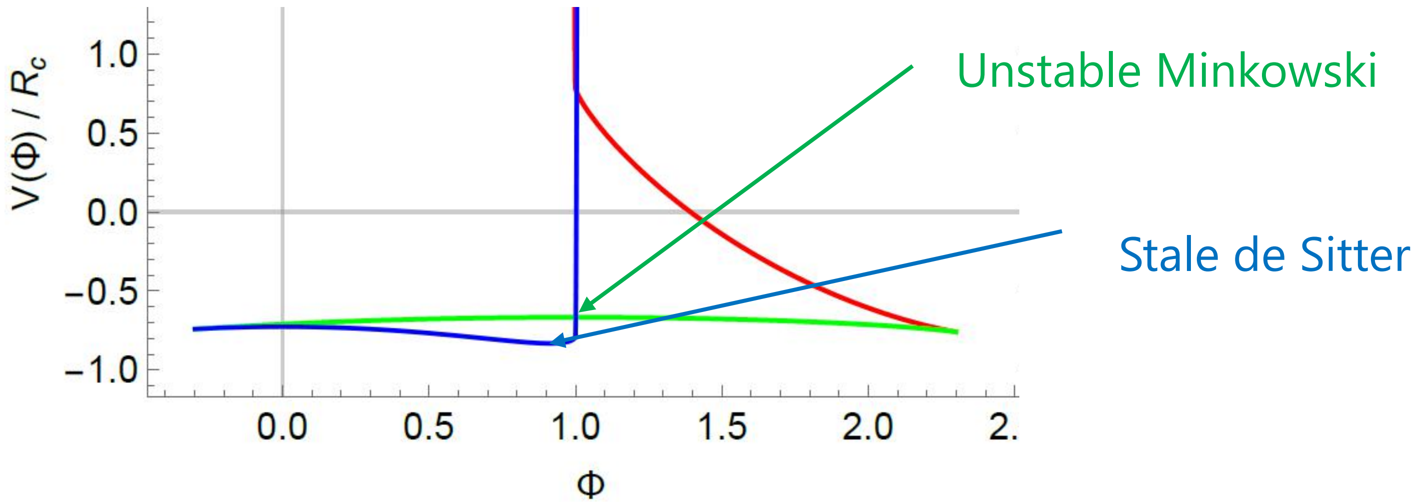
$$\left(\Box - m_\Phi^2\right)\varphi = -\frac{1}{3}\Phi_{\min} h^{\mu\nu} R_{\mu\nu}^{(b)} + \frac{1}{6} F(R^{(b)}) h$$

Input background curvature

Cf.) Mass >> Dark Energy

Ignore background curvature

Cf.) Wavelength << Hubble



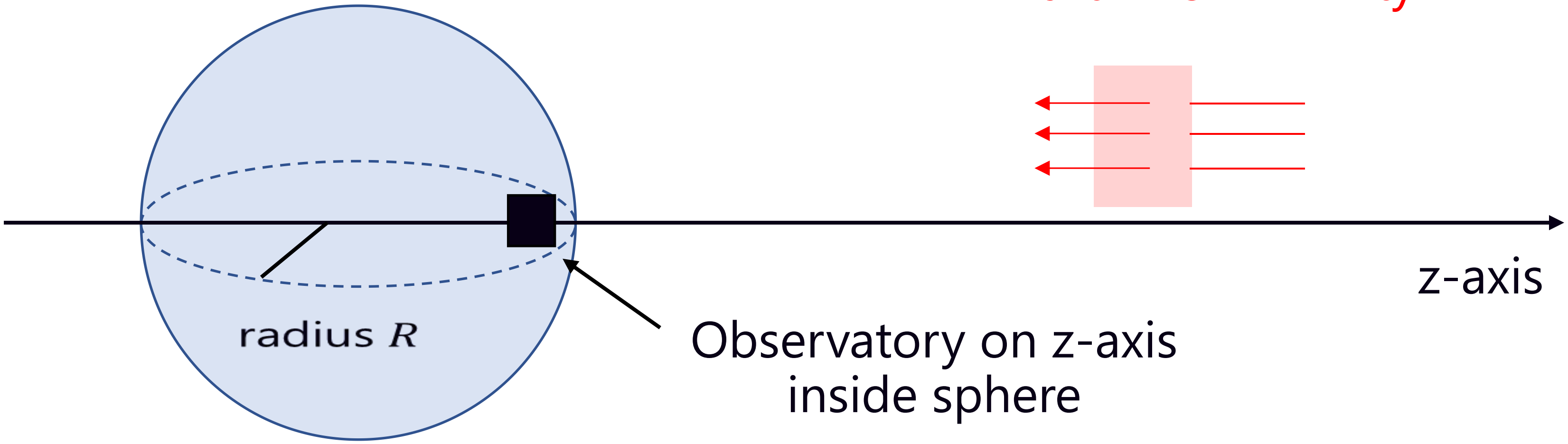
To Consider Ground-Based GW Observatory

We solve KG-type equation

$$\left(\Box - m_{\Phi}^{2}\right)\varphi = 0$$

Following simplified environment:

Earth = High-density region
= constant density ρ



More Simplification…

If Scalar Waves Vertically Enter Near Z-axis

- To ignore curvature. of sphere (Boundary is flat)
- Penetrating waves in high-density region = Plane waves

$$\left(-\frac{\partial^2}{\partial t^2}+\frac{\partial^2}{\partial z^2}\right)\phi(t,z)=m_\Phi^2(z)\phi(t,z)$$

Fourier Transformation

$$\phi(t,z)=\int\frac{\mathrm{d}\omega}{2\pi}\,\tilde{\phi}(\omega,z)\,e^{-i\omega t}\quad\Rightarrow\quad\frac{\mathrm{d}^2\tilde{\phi}}{\mathrm{d}z^2}=\left[m_\Phi^2(\Phi_{bg})-\omega^2\right]\tilde{\phi}$$

Suppression of Scalar Mode

(1+1)-dim. Wave Equation

Inside sphere, low frequency $\omega < m_\Phi$

$$\tilde{\phi} = C\, e^{\sqrt{m_\Phi^2 - \omega^2} z}$$

If $\omega < m_\Phi$, scalar mode is decaying!

N.B.) High frequency $\omega > m_\Phi$, only phase changes

High-density region = Atmosphere of Earth

$$\rho = 10^{-9}\ [\mathrm{g/cm^3}] \text{ @ altitude } |z| = 10^5 [\mathrm{m}].$$

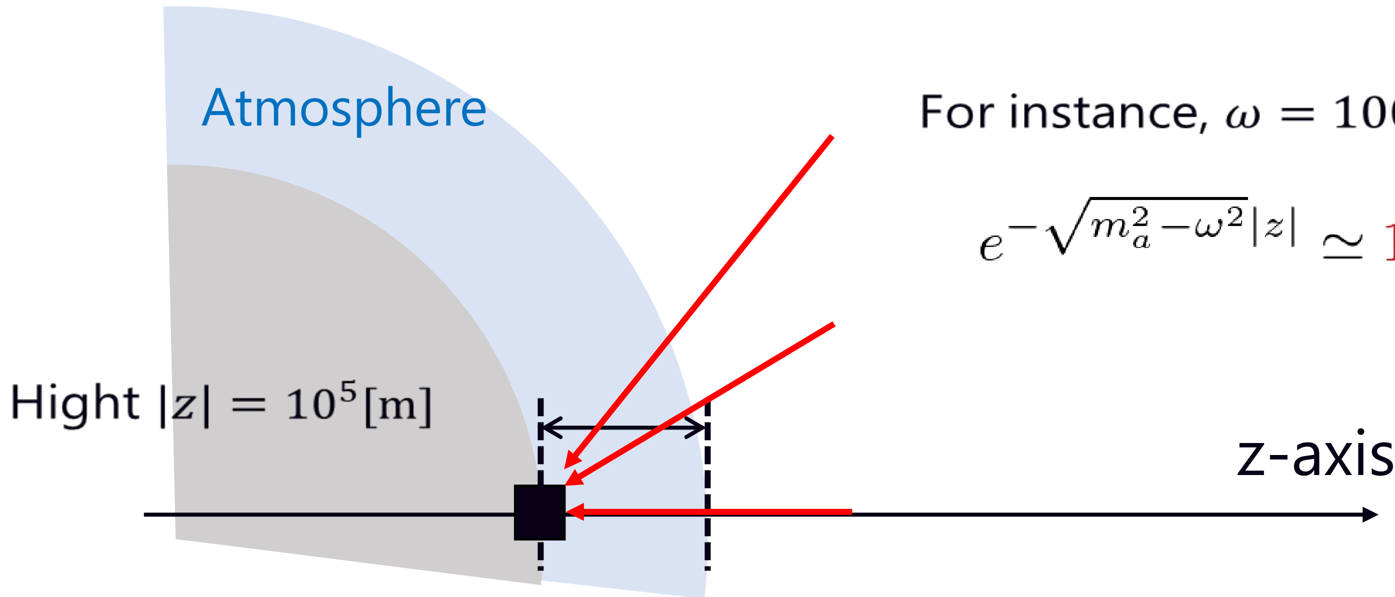
Other parameters,

$$\beta = 2,\ n = 1,\ \alpha = 2 \times 10^{22}\ [\mathrm{GeV^{-2}}]$$

Criterion frequency $\omega_c \simeq 1 \times 10^{13} [\mathrm{Hz}]$

How much decaying?

- Vertically entering wave = shortest path
- Damping factor in realistic situation?



For instance, $\omega = 100$ [Hz]

$$e^{-\sqrt{m_a^2-\omega^2}|z|} \simeq 10^{-6\times 10^9}$$

Non-vertically entering waves

- They travel longer path ($|z| \gg 10^5[\mathrm{m}]$)
- Further dumping by chameleon mechanism

Summary and Conclusion

Scalar mode of GWs

- Formulation with matter for chameleon Mechanism
- Maximally symmetric space-time for simplification

Chameleon mechanism in GWs

- Toy-model to reproduce environment of ground-based GW observatory
- Decaying due to chameleon mechanism (as we expected!?)

Observation of scalar mode

- Impossible by ground-based observatory
- Maybe possible by space-based observatory due to low-energy density?
- Test environment-dependence of GW signal